

Nuclear Matter Properties of the Modified Quark Meson Coupling Model

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Abstract

We explore in more detail the modified quark meson coupling (MQMC) model in nuclear matter. Based on previous studies two different functional forms for the density dependence of the bag constant are discussed. For uniform matter distributions the MQMC model can be cast in a form identical to QHD by a redefinition of the sigma meson field. It is then clear that modifications similar to those introduced in QHD will permit the reproduction of all nuclear matter properties including the compressibility. After calibrating the model parameters at equilibrium nuclear matter density, the model and parameter dependence of the resulting equation of state is examined. Nucleon properties and scaling relations between the bag constant and the effective nucleon mass are discussed.

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I. INTRODUCTION

The description of the nuclear many-body problem in terms of strongly interacting quarks and gluons is one of the major challenges in nuclear physics. At present, however, rigorous studies of QCD are restricted to matter systems at high temperature and zero baryon density. Because of the nonperturbative features, it appears very difficult to derive from this theory predictions for processes at energy scales relevant for low- and medium-energy nuclear phenomenology.

On the other hand, it is well known that despite these difficulties nuclear phenomenology can be efficiently described using hadronic degrees of freedom. While this framework has been very successful in describing the features of nuclear matter and the binding energy systematics of finite nuclei, experiments, such as deep inelastic scattering off nucleons, provide evidence that the standard hadronic picture has to be corrected. For example, the prominent EMC effect which reveals medium modifications of the internal structure of the nucleon [1].

Moreover, hadronic models are often extrapolated into regimes of high density and temperature to extract the nuclear equation of state, which is the basic ingredient in many astrophysical applications and in microscopic models of energetic nucleus–nucleus collisions. One can expect that under these extreme conditions quark degrees of freedom become important.

To address these issues it is necessary to build theories which incorporate quark-gluon degrees of freedom and which help to bridge the gap between nuclear phenomenology and the underlying physics of strong interactions. An important criteria for these new models is that they reproduce results based on the established hadronic framework.

Guided by the symmetry breaking pattern of QCD, much effort has recently been devoted to the study of effective Lagrangians for low-energy strong interactions. A typical example is the Nambu and Jona-Lasinio (NJL) model [2]. These models are mainly concerned with the spontaneous breaking and restoration of chiral symmetry but it is not clear if basic

features of nuclear phenomenology, such as saturation of nuclear matter, can be described properly. On the other hand, almost a decade ago, Guichon [3] proposed a quark-meson coupling (QMC) model in which nucleons arise as MIT bags interacting through meson mean fields. This model was refined later by including center-of-mass corrections [4] and applied to nuclear matter [5–12] and also, more recently, to finite nuclei [13,14]

Although it provides a simple and attractive framework to describe nuclear systems in terms of quark degrees of freedom, the QMC model has a serious shortcoming. It predicts much smaller scalar and vector potentials than obtained in successful hadronic models [11,12]. As a consequence the nucleon mass is much too high [5,6,11,12] and the spin-orbit force is too weak to explain spin-orbit splittings and spin observables in finite nuclei.

A well established framework for relativistic hadronic models is provided by quantum hadrodynamics (QHD) [15]. Numerous calculations have established that relativistic mean-field models based on QHD provide a realistic description of the bulk properties of finite nuclei and nuclear matter [15]. One of the key observations in their success is that nucleon propagation in the nuclear medium is described by a Dirac equation featuring large scalar and vector potentials. They emerge from the interaction of a nucleon with all other nucleons in the Fermi sea via the exchange of isoscalar scalar and vector mesons.

Recently, it was pointed out that the small vector and scalar potentials in the QMC model are due to the assumption that the bag constant does not change in the nuclear environment [11,12]. By introducing a density dependent bag constant it was demonstrated that large scalar and vector potentials can be produced. A necessary condition is that the value of the bag constant in the nuclear environment significantly drops below its free-space value. As a consequence relativistic nuclear phenomenology can be recovered from a modified quark-meson coupling (MQMC) model [11].

The central issue of the MQMC model is the density dependence of the bag constant. A priori this is not known and the idea of the MQMC model is to parametrize the bag constant and to determine the parameters by calibrating to observed nuclear properties. Two different model types have been proposed [11,12]: a direct coupling model in which the bag constant

is a function of the scalar field and a scaling model in which the bag constant is related to the effective nucleon mass. The density dependence is then generated self-consistently in terms of these in-medium quantities. However, the proposed models are not flexible enough to predict nuclear matter properties on a satisfactory level. The trend in the MQMC model is to predict reasonable values for the effective nucleon mass but compressibilities which are too high and vice versa [11,12].

We adopt the approach of Refs. [11,12] with the goal of generalizing the proposed models. We assume two different functional forms for the bag constant. In one case it depends on the scalar field only; in our second model we study the bag constant as a function of the effective nucleon mass. To provide sufficient flexibility we model the functional form by using polynomial and Padé parametrizations. The unknown coefficients serve as our model parameters. We investigate how well the models can be calibrated by fitting the parameters to properties of nuclear matter, which we take to be: the equilibrium density and binding energy ($\rho_N^0, -e_0$), the nucleon effective (or Dirac) mass at equilibrium ($M_{N,0}^*$) and the compression modulus (K_0). In general the models contain more parameters than there are normalization conditions. Thus families of models can be generated which describe exactly the same nuclear matter properties at equilibrium. This allows us to study different physical situations and to search for model dependence in the predictions.

Because the QMC model was proposed to describe “new” physics beyond the standard hadronic picture in nuclear matter and finite nuclei, we address the crucial question if the model is consistent with established results. As a first step we investigate the connection between the MQMC model and QHD. We demonstrate that in nuclear matter the MQMC energy functional is formally equivalent to the expression obtained in QHD with a general nonlinear scalar potential. The quark substructure is entirely contained in the scalar potential and, in principle, this provides a tool to generate hadronic potentials based on a quark model. The explicit form of the hadronic potential depends on the model which is employed for the bag constant. Thus determining the parameters on the quark level is equivalent to calibrating the potential on the hadronic level which is a well established procedure in

nuclear matter calculations [16,17].

We apply our model to symmetric nuclear matter and compare the predictions with QHD mean-field calculations. We employ a version of QHD which contains quartic and cubic scalar self-interactions [18] and which is calibrated to reproduce the same nuclear matter properties as the MQMC model. We show that different parametrizations and models for the bag constant lead to equivalent nuclear properties at low and moderate densities. In this region the MQMC model predictions are in excellent agreement with QHD. In contrast, the original version of the QMC model leads to substantially different results. Hence we are lead to the satisfactory conclusion that our generalized MQMC model predicts nuclear matter properties of the same quality as other established mean-field models. The key to this success is the correlation between the bag constant and nuclear matter properties. Confirming the results of Refs. [11,12], we find that the bag constant has to be significantly smaller than its free-space value to reproduce the desired effective nucleon mass at equilibrium. More importantly, the explicit value of the bag constant at equilibrium is model independent, *i.e.* independent of the details of the parametrization.

The exact density dependence of the bag constant is essentially unknown. However, based on general theoretical arguments scaling relations among in-medium quantities have been proposed in the recent literature [19,20]. Since our approach establishes a direct connection between nuclear phenomenology and the quark substructure we investigate to what extent the proposed scaling relation between the bag constant and the effective nucleon mass is realized; at this point we encounter a clear model dependence. Although all our models are calibrated to produce identical nuclear matter properties we can build models which exactly follow the proposed scaling relation but also models which differ considerably. A similar model dependence can be observed for the bag radius. In accordance with previous studies [11,12] we find a sizable increase of the bag radius as a consequence of the decreasing bag constant. However, the quantitative predictions exhibit a strong model and parameter dependence.

The outline of this paper is as follows: In Sec. II, we give a short description of the QMC

model and summarize the relations which determine the nuclear matter properties. Section III is devoted to discuss the connection between the QMC model and QHD. We also discuss the calibration procedure. In Sec. IV, we apply our model to symmetric nuclear matter. We compare our results with QHD and with the original version of the QMC model. Section V contains a short summary and our conclusions.

II. THE QUARK-MESON COUPLING MODEL

In this section, we briefly summarize the relations which determine the nuclear equation of state in the quark-meson coupling model. For further details we refer the reader to Refs. [6,11,12].

In the QMC model the nucleon in the nuclear medium is described as a static, spherical MIT bag in which quarks couple to meson mean fields. In symmetric matter they are taken to be neutral scalar (σ) and vector fields (V^μ).

The energy of a bag consisting of three quarks in the ground state can be expressed as

$$E_{bag} = 3\frac{\Omega_q}{R} - \frac{Z}{R} + \frac{4}{3}\pi R^3 B . \quad (1)$$

where the parameter Z accounts for the zero point motion and B is the bag constant. The coupling of the quarks to the scalar field is inherent in the quantities Ω_q and x which are given by

$$\begin{aligned} \Omega_q &= \sqrt{x^2 + (Rm_q^*)^2} \\ j_0(x) &= \left(\frac{\Omega_q - Rm_q^*}{\Omega_q + Rm_q^*} \right)^{1/2} j_1(x) , \end{aligned} \quad (2)$$

and where $m_q^* = m_q^0 - g_\sigma^q \sigma$ denotes the effective quark mass and m_q^0 is the current quark mass. For simplicity we work in the chiral limit, *i.e.* $m_q^0 = 0$.

After correcting for the spurious center of mass motion, the effective mass of a nucleon bag is given by [4]

$$M_N^* = \sqrt{E_{bag}^2 - 3x^2/R^2} . \quad (3)$$

For a fixed meson mean-field configuration the bag radius R is determined by the equilibrium condition for the nucleon bag in the medium

$$\frac{\partial M_N^*}{\partial R} = 0 . \quad (4)$$

In free space M_N can be fixed at its experimental value 939 MeV and the condition Eq. (4) to determine the parameters $B = B_0$ and $Z = Z_0$. For our choice, $R_0 = 0.6$ fm, the result for $B_0^{1/4}$ and Z_0 are 188.1 MeV and 2.03, respectively.

The total energy density of nuclear matter can be written as [4,6]

$$\mathcal{E}_{QMC} = \frac{\gamma}{2\pi^2} \int_0^{k_F} dk k^2 (k^2 + M_N^{*2})^{1/2} + g_v V_0 \rho_N - \frac{1}{2} m_v^2 V_0^2 + \frac{1}{2} m_s^2 \sigma^2 . \quad (5)$$

In symmetric matter ($\gamma = 4$) the Fermi momentum of the nucleons is related to the conserved baryon density by

$$\rho_N = \frac{\gamma}{6\pi^2} k_F^3 . \quad (6)$$

The meson mean fields are determined by the general thermodynamic condition that they should make the energy per nucleon stationary. For the time-like component of the vector field this leads to the relation

$$g_v V_0 = \frac{g_v^2}{m_v^2} \rho_N , \quad (7)$$

whereas the scalar field is determined by the self-consistency equation

$$\sigma = \frac{C_q(\sigma)}{m_s^2} \frac{\gamma M_N^*}{2\pi^2} \int_0^{k_F} dk \frac{k^2}{(k^2 + M_N^{*2})^{1/2}} . \quad (8)$$

The details of the quark substructure are entirely contained in the effective coupling $C_q(\sigma)$ which is related to the effective nucleon mass by

$$C_q(\sigma) = -\frac{\partial M_N^*}{\partial \sigma} , \quad (9)$$

and which depends on the explicit form of the bag constant. Details can be found in Refs. [11,12].

III. GENERATING NONLINEAR MEAN-FIELD MODELS FROM MODIFIED QUARK-MESON COUPLING MODELS

In the original version of the QMC model [3,4,6] the bag parameters B and Z were held fixed at their free space values $B = B_0, Z = Z_0$. The bag constant B is a nonuniversal quantity associated with the QCD trace anomaly. In the nuclear environment it is expected to decrease with increasing density as argued in Ref. [20]. At present, however, no reliable information on the medium dependence of B is available on the level of QCD calculations. On the other hand, much effort has recently been devoted to the study of effective models which approximate low energy QCD. The guiding principle for constructing such effective models are the symmetries and the symmetry breaking patterns of QCD, in particular chiral symmetry breaking. Typical representatives are Nambu and Jona-Lasinio (NJL) models [2] and related chiral meson lagrangians [21]. In this framework attempts have been made to model the QCD trace anomaly by introducing a scalar glueball field [22]. The concept of a bag constant arises naturally in these models and it is a common feature to predict a decreasing value of B when the density (or temperature) of the nuclear environment is increased [23].

To account for this physics in the QMC approach two different models for the bag constant have been proposed [11,12]. A direct coupling between the bag constant and the scalar mean field

$$\frac{B}{B_0} = \left[1 - g_\sigma \frac{4}{\delta} \frac{\sigma}{M_N} \right]^\delta, \quad (10)$$

and a scaling model which relates the bag constant in the medium directly to the effective nucleon mass

$$\frac{B}{B_0} = \left[\frac{M_N^*}{M_N} \right]^\kappa. \quad (11)$$

Thus, rather than focus on the calculation of the bag constant in the medium the idea of the modified QMC model is to parametrize the density dependence in terms of in-medium

quantities. A similar approach is used to construct nonlinear mean-field models in quantum hadrodynamics (QHD) where the unknown density dependence of the nuclear energy functional is parametrized by nonlinear meson-meson interactions [16,24]. This task is certainly more difficult on the quark level. The direct coupling model is inspired by NJL type nontopological soliton models for the nucleon [25], where a scalar soliton field is responsible for the binding of three quarks to form a nucleon. On the other hand, the scaling model is related to the idea of a general scaling relation between in-medium quantities as proposed by Brown and Rho [19,20].

Generally, the parameter Z may also be modified in the medium [26]. However, in contrast to the bag constant, there is less physical intuition how the medium dependence of Z can be cast in a model. Here we assume that the density dependence of Z can be disregarded and take $Z = Z_0$.

The original QMC model with $B = B_0$ has a serious shortcoming. The predicted mean fields are much smaller than obtained in established relativistic mean-field models [16,27–30]. As a consequence, the effective nucleon mass is much too big [5,6]. Recently, it has been demonstrated that this shortcoming can be significantly corrected and that relativistic nuclear-phenomenology can be recovered from the models given by Eq. (10) and Eq. (11) [11,12]. However, the systematics of the predicted nuclear matter properties is still not satisfactory. The MQMC model produces acceptable values for the effective nucleon mass but values for the compressibility which are too high and vice versa [11,12]. In our work we generalize the direct coupling model and the scaling model demonstrating that the resulting improved MQMC predicts nuclear matter properties of the same quality as in other successful relativistic mean-field models ¹.

Furthermore, we will show that the relation between the QMC model and QHD is more

¹Experience has shown that an accurate reproduction of nuclear matter properties leads to realistic results when the calculations are extended to finite nuclei [16,27–30]

general and direct than previously thought [6,12]. This will allow us a more consistent comparison between these different approaches.

We start the analysis with a comparison of the energy density in Eq. (5) with the corresponding expression in QHD [15]

$$\mathcal{E}_{QHD} = \frac{\gamma}{2\pi^2} \int_0^{k_F} dk k^2 (k^2 + M_N^{*2})^{1/2} + \frac{g_v^2}{2m_v^2} \rho_N^2 + U_s(\phi) , \quad (12)$$

where the effective nucleon mass is given by $M_N^* = M_N - g_s \phi$. The standard form of the nonlinear scalar potential is

$$U_s(\phi) = \frac{1}{2} m_s^2 \phi^2 + \frac{\kappa}{6} \phi^3 + \frac{\lambda}{24} \phi^4 . \quad (13)$$

Although the cubic and quartic terms provide sufficient flexibility for an accurate calibration of the nuclear equation of state [18], we will assume a general functional form for the potential. The scalar field is determined by the self-consistency equation

$$\frac{\partial U_s(\phi)}{\partial \phi} = g_s \frac{\gamma M_N^*}{2\pi^2} \int_0^{k_F} dk \frac{k^2}{(k^2 + M_N^{*2})^{1/2}} . \quad (14)$$

The equivalence of the two sets of equations, Eqs. (5) and (8), and Eqs. (12) and (14), can be demonstrated by performing a redefinition of the scalar field in the QMC model

$$g_0 \phi(\sigma) \equiv M_N - M_N^*(\sigma) = M_N - \sqrt{E_{bag}^2 - 3x^2/R^2} . \quad (15)$$

The transformation does not depend explicitly on the density if we assume

$$B = B(\sigma, M_N^*) ,$$

as suggested in Eqs. (10) and (11). The coupling g_0 is chosen to normalize the new field according to

$$\phi(\sigma)_{\sigma \rightarrow 0} = \sigma + O(\sigma^2) ,$$

and it is given by

$$g_0 = - \left. \frac{\partial M_N^*(\sigma)}{\partial \sigma} \right|_{\sigma=0} . \quad (16)$$

The transformation is well defined provided that the effective mass changes monotonically with the scalar field. Inverting the relation Eq. (15) leads to a nonlinear scalar potential in the energy density Eq. (5)

$$\frac{1}{2}m_s^2\sigma^2(\phi) \equiv U_s(\phi) . \quad (17)$$

At this point the energy density in the QMC model is *identical* to the QHD expression Eq. (12) with a general scalar potential. For finite nuclei the transformation also introduces a change in the kinetic energy of the scalar meson. This will mainly lead to a different description of the surface energy, but this effect is expected to be small.

As shown in Ref. [12] a significant simplification arises for models with $B = B(\sigma)$ if $g_\sigma^q = 0$, *i.e.* if there is no direct coupling of the quarks to the scalar field. In this case one obtains the scaling relation [12]

$$\frac{B}{B_0} = \left(\frac{M_N^*}{M_N} \right)^4 . \quad (18)$$

Thus, if the bag constant is of the specific form

$$\frac{B}{B_0} = \left(1 - g_B \frac{\sigma}{M_N} \right)^4 , \quad (19)$$

the transformation Eq. (15) is linear leading to the original Walecka model [32] with a quadratic potential

$$U_s(\phi) = \frac{1}{2}m_s^2\phi^2 . \quad (20)$$

The details of the quark substructure are entirely contained in the nonlinear potential $U_s(\phi)$. This implies that if the bag constant, more generally the bag parameters, was known as a function of the scalar field and the effective mass, the steps leading to Eq. (17) would permit the *prediction* of potentials for hadronic mean-field models.

On the other hand starting with a known potential, Eq. (17) defines the inverse field transformation to Eq. (15)

$$\sigma(\phi) \equiv \frac{\sqrt{2U_s(\phi)}}{m_s} . \quad (21)$$

After substituting this result in Eq. (12), the QHD expression for the energy is exactly of the same form as in the MQMC model. The self-consistency condition Eq. (14) changes to

$$\sigma = \frac{C(\sigma)}{m_s^2} \frac{\gamma M_N^*}{2\pi^2} \int_0^{k_F} dk \frac{k^2}{(k^2 + M_N^{*2})^{1/2}} , \quad (22)$$

where the coupling is given by

$$C(\sigma) = g_s m_s \frac{\sqrt{2U_s}}{\frac{\partial U_s}{\partial \phi}} . \quad (23)$$

Thus, the MQMC model is formally equivalent to a nuclear mean field model with a *field dependent* scalar-nucleon coupling. Field dependent nucleon-meson couplings are often interpreted as an indication for the compositeness of the nucleon [6,13] but from a modern point of view they appear naturally in hadronic models as a result of field redefinitions. In our case the field dependent coupling is equivalent to a subset of nonlinear meson self-interactions ².

Using the relation between MQMC and QHD one can determine the bag constant as a function of σ . This approach is useful since it establishes a direct connection between nuclear phenomenology and the medium dependence of the bag parameters.

To be more specific let us assume $B = B(\sigma)$ and $g_\sigma^q = 0$. The scaling relation Eq. (18) together with the expression for the effective mass in QHD

$$M_N^* = M_N - g_s \phi(\sigma) \quad (24)$$

then leads to

$$\frac{B}{B_0} = \left(1 - \frac{g_s \phi(\sigma)}{M_N} \right)^4 . \quad (25)$$

In general, the transformation which relates the two scalar fields is not known, but Eq. (25) motivates the *ansatz*

$$\frac{B}{B_0} = \left(1 - g_B \frac{\sigma}{M_N} F(\sigma) \right)^\kappa \quad \text{with} \quad F(0) = 1 , \quad (26)$$

² For a discussion of the equivalence between field dependent meson-nucleon couplings and nonlinear meson self-interactions in the framework of QHD see Ref. [15]

which includes the original form Eq. (10).

Similarly, the scaling model can be generalized by

$$\frac{B}{B_0} = \left(\frac{M_N^*}{M_N} \right)^\kappa G(M_N^*) \quad \text{with} \quad G(1) = 1 . \quad (27)$$

We studied polynomial and Padé parametrizations for the unknown functions F and G which all lead to qualitatively similar results at low and moderate densities. In the following we will present results obtained by using a simple polynomial

$$F(\sigma) = 1 + \alpha\sigma + \beta\sigma^2 , \quad (28)$$

for the generalization of the direct coupling model, and a Padé form

$$G(M_N^*) = \frac{1}{1 - a - b - c + aM_N^* + bM_N^{*2} + cM_N^{*3}} , \quad (29)$$

for the generalization of the scaling model. For notational convenience we will refer to the model in Eq. (26) as MQMC_A and to the model in Eq. (27) as MQMC_B.

Since the parameters B_0 and Z are fixed to reproduce the nucleon mass in the vacuum our models contain six free parameters. The parametrization of the bag constant contains the parameter κ and the three couplings (g_B, α, β) and (a, b, c) for MQMC_A and for MQMC_B, respectively; in addition values for the quark-meson couplings g_σ^q and for the ratio g_ν/m_ν are needed³. Four of the six parameters can be chosen to reproduce the equilibrium properties of symmetric nuclear matter, which we take as the equilibrium density and binding energy $(\rho_N^0, -e_0)$, the nucleon effective mass at equilibrium $(M_{N,0}^*)$ and the compression modulus (K_0) . The first three of these are tightly constrained [30], whereas the latter is not. The set of equilibrium properties used here [16] are listed in Table I; these are motivated by successful descriptions of bulk and single-particle nuclear properties [16,30,31]. Since there are more

³Strictly speaking, in nuclear matter only the ratios $(g_B/g_\sigma^q, \alpha/g_\sigma^q, \beta/g_\sigma^{q2}, g_\sigma^q/m_\sigma, g_\nu/m_\nu)$ are relevant for the calibration procedure. In order to vary the scalar coupling the mass of the scalar meson needs to be fixed. We choose $m_\sigma = 550$ MeV.

free parameters than constraints, we proceed as follows. We choose values for the coupling g_σ^q and for κ and determine the remaining couplings by requiring that they reproduce the desired equilibrium properties. This is achieved by solving a set of transcendental equations that relate the parameters directly to the nuclear matter properties; this is a well established procedure in nuclear matter calculations [16,17]. The original version of the QMC model with $B = \text{const.}$ contains only two parameters, g_σ^q and g_v/m_v , which are usually chosen to reproduce the binding energy and the density at nuclear matter equilibrium. In our discussion we use the parameters given in Ref. [6]. For comparison we employ the QHD model with the nonlinear potential in Eq.(13). The parameters are determined to reproduce the same equilibrium properties as in the QMC model.

Our primary goal is to study the influence of the in-medium bag constant on the equation of state (EOS) of nuclear matter. The freedom of varying the parameter κ will allow us to investigate scaling relations between the bag constant and the effective nucleon mass. Moreover, models with $g_\sigma^q = 0$ and $g_\sigma^q \neq 0$ describe different physical situations. For $g_\sigma^q = 0$ the scalar meson couples only to the surface of the bag. Apart from an overall shift in the single-particle energies, due to the vector-meson mean field, the properties of the quarks inside the bag are not changed by the nuclear medium. In contrast, the quarks acquire a density dependent effective mass $m_q^* = m_q^0 - g_\sigma^q \sigma$ for nonvanishing couplings g_σ^q .

A decreasing effective nucleon mass in the original QMC model with $B = B_0$ requires a positive quark-meson coupling ($g_\sigma^q > 0$) which implies a negative effective quark mass. A similar effect can be observed in soliton models [25]. Close to the origin of the soliton the attractive scalar potential leads to a negative effective quark mass. But here this is less disturbing since the effective mass depends on the space coordinates and becomes positive in the outer region of the soliton. In the MQMC it is possible to chose $g_\sigma^q > 0$ or $g_\sigma^q < 0$ and to generate positive or negative effective quark masses. However, we observe that the sign of the effective quark mass has no impact on the properties of nuclear matter. Models with either sign are qualitatively equivalent.

A priori we have no specific guidance on the allowed values of κ and g_σ^q . We observe that

not all possible choices permit the reproduction of the desired equilibrium properties. For example, MQMC_B has no solution for $g_\sigma^q \lesssim 0.8$. We analyzed MQMC_A for $3 \lesssim \kappa \lesssim 7$ and $0 \leq g_\sigma^q \lesssim 2$, and MQMC_B for $3 \lesssim \kappa \lesssim 5$ and $0.8 \lesssim g_\sigma^q \lesssim 1.5$.

IV. NUCLEAR MATTER PROPERTIES

The density dependence of the nucleon mass is controlled by the effective coupling $C_q(\sigma)$ in the self-consistency equation Eq. (8). For MQMC_A this quantity is indicated in Fig. 1. To normalize the curves at the origin we divided $C_q(\sigma)$ by g_0 which was defined in Eq. (16). The effective coupling for QHD, given by Eq. (23), and for the original QMC model are also indicated. We emphasize that all parametrizations except the original QMC model reproduce the same equilibrium properties listed in Table I. For small values of the scalar field MQMC_A and QHD are in good agreement predicting nearly constant effective couplings. In contrast, the curve for the QMC model steadily decreases and eventually becomes negative.

The consequences for the effective nucleon mass can be studied in Fig. 2. As expected from Fig. 1 the masses for MQMC_A and for QHD are nearly identical up to 1.5 nuclear matter densities. For $B = B_0$ the effective mass decreases very slowly and, when the point $C_q(\sigma) = 0$ is reached, increases again. The curves terminate at some maximum density, which corresponds to a maximum value of the scalar field. For $g_\sigma^q = 0$ this occurs at $B = M_N^* = 0$. At that point the radius diverges. The curves for $g_\sigma^q \neq 0$ terminate when x , the solution of Eq. (2), approaches zero.

Fig.3 shows the effective nucleon mass for MQMC_B. Similar to what we observed for MQMC_A, the curves are nearly indistinguishable from the QHD result at low and moderate densities. At higher densities MQMC_B leads to a slowly decreasing effective mass which approaches a nonzero asymptotic value.

Fig.4 indicates the predicted nonlinear potentials in Eq. (17) as a function of the transformed scalar field given by Eq. (15). The value $g_0\phi/M_N = 0.4$ corresponds to the saturation point of nuclear matter. For $g_0\phi/M_N < 0.6$ the predicted MQMC potentials are almost iden-

tical to the nonlinear QHD potential in Eq. (13). In contrast the potential in the QMC model is much smaller. Here the saturation point is at $g_0\phi/M_N = 0.11$.

Due to the calibration procedure, the very good agreement of the different models and parametrizations at low and moderate densities is certainly not surprising. As clearly visible in Figs. 1-4, the predictions of MQMC_A and MQMC_B vary considerably from QHD above the saturation point of nuclear matter. We also observe that different model types and different parametrizations within a specific model are no longer equivalent at high densities. This is certainly an indication that an extrapolation into the regime of high densities might be problematic.

In Fig. 5 we show the binding energy curves of symmetric matter for MQMC_A. The stiffness of the EOS is controlled by the parameters κ and g_σ^q . The curves become softer for smaller values of κ and stiffer if the quark-meson coupling is increased [11,12]. Also indicated in Fig. 5 is the binding energy curve of the original QMC model. Although the compression modulus is only slightly lower ($K_0 = 223\text{MeV}$) than in the other models ($K_0 = 250\text{MeV}$) the EOS in the original QMC model is substantially softer. To understand this difference, it is useful to split the binding energy according to

$$e_0 = \frac{\mathcal{E}}{\rho_N} - M_N = \frac{\mathcal{E}^0}{\rho_N} + \frac{U_v}{\rho_N} + \frac{U_s}{\rho_N} , \quad (30)$$

with

$$\begin{aligned} \mathcal{E}^0 &= \frac{\gamma}{2\pi^2} \int_0^{k_F} dk k^2 (k^2 + M_N^{*2})^{1/2} - M_N \rho_N , \\ U_v &= \frac{g_v^2}{2m_v^2} \rho_N^2 , \\ \text{and } U_s &= \frac{1}{2} m_s^2 \sigma^2 . \end{aligned}$$

In Fig. 6 the three different contributions are indicated as a function of the density. Saturation arises as a competition between the decreasing effective mass which lowers the kinetic energy and the increase of the energy due to the increasing density. As expected, the curves for QHD and MQMC_A are almost identical. The quantitative difference to the original QMC model is striking. To achieve saturation at a very high effective mass ($M_N^* = 0.89$) only a

small contribution of the vector part U_v , *i.e.* a small coupling g_v^2/m_v^2 , is required. At high densities the term U_v dominates and a smaller coupling leads to a softer EOS. This also explains the tendency of the QMC model to produce small vector mean fields [11,12].

The in-medium bag constant as a function of the effective nucleon mass is indicated in Fig. 7. We consider MQMC_A in part (a) and MQMC_B in part (b). To reproduce the desired effective nucleon mass at the saturation point, B is required to decrease substantially below its free-space value. The values of B/B_0 are between the two lines $(M_N^*/M_N)^4$ and $(M_N^*/M_N)^3$. As mentioned in the last section, in the MQMC_A model a special situation arises for $g_\sigma^q = 0$. In that case the bag constant scales as $(M_N^*/M_N)^4$ for all values of κ [12].

We emphasize that all curves produce identical nuclear matter properties. Hence there is no compelling evidence from nuclear phenomenology that the bag constant should scale with a specific power κ , *e.g.*, like $(M_N^*/M_N)^4$ as proposed in Ref. [19]. The tendency of the MQMC approach is to predict values of B which are slightly higher. This becomes more apparent in Fig. 8 where the bag constant at the saturation point as a function of g_q^σ is indicated. We examine MQMC_A for three different values of the nucleon mass at equilibrium. The value $g_q^\sigma = 0$ correspond to the scaling behavior $B/B_0 = (M_N^*/M_N)^4$. Because the calibration procedure determines the value of B , the curves are *model independent*, *i.e.* they depend only on the coupling but they are independent of the functional form of $B = B(\sigma)$. The curves in Fig. 8 are also independent of the compressibility which merely determines the derivatives of B with respect to the scalar field in MQMC_A and the derivative with respect to the effective nucleon mass in MQMC_B. The key quantity here really is the effective nucleon mass.

At this point, some caveats concerning the comparison with other approaches must be added. As mentioned earlier, the prediction of a reduced bag constant in the nuclear environment is also a common feature in effective models for low energy QCD, *e.g.* chiral models. Since none of these models can strictly be derived from QCD it is not clear to what extent results can be compared. Within the MQMC approach, it was one of our goals to demonstrate that it is possible to obtain nuclear matter results which are of the same quality

as in any other successful hadronic mean-field model. In contrast, chiral models are more concerned with the description of the underlying physics, *i.e.* the breaking and restoration of chiral symmetry. To the best of our knowledge, it has not been demonstrated whether it is possible to describe nuclear saturation and properties of finite nuclei on that level.

The corresponding bag radius is indicated in Fig. 9. As a consequence of the rapidly decreasing bag constant the MQMC models predict a picture of a significantly "swollen" nucleus [11,12]. This effect is most drastic in MQMC_A for vanishing quark-meson couplings. As mentioned above, the effective nucleon mass and therefore also the bag constant vanishes at some finite value of the density. Consequently, the radius diverges at this point. Although all the models produce identical nuclear matter properties near equilibrium they yield significantly different radii, even at low densities. Also shown in Fig. 9 is the curve $R_c = (3/4\pi\rho_N)^{1/3}$ which indicates the "critical" radius where the individual bags start overlapping, signaling the breakdown of the simple bag model. In our model this occurs slightly above the saturation density. This behavior is in sharp contrast to the original QMC model which predicts a nearly constant radius.

We emphasize that it is not possible to achieve reasonable nuclear matter properties and a small radius at the same time because the calibration procedure determines the value of the bag constant. As indicated in Fig. 8, the acceptable range of $M_N^*/M_N = 0.6 - 0.7$ always requires a small value of B which in turn leads to a large radius. This is certainly a dilemma. On one hand we have demonstrated that the MQMC model can be improved and calibrated so that nuclear matter properties are accurately reproduced. The most important quantity here is the effective nucleon mass which is tightly constrained by nuclear observables [30]. On the other hand we found that the required bag constant in the medium leads to radii which are unreasonably high. Furthermore the radii are very sensitive to the model features and parametrizations. The size of a nucleon in matter is certainly a subtle concept and one might argue that physical observables do not depend on the bag radius [33]. However, it is an important phenomenological quantity in many nuclear physics issues where bag models are employed to describe features which depend on the intrinsic structure of the nucleon (

see for example [34]).

Moreover, one motivation for generating the nuclear equation of state on basis of a quark model is the hope that such an approach is well suited to describe systems at high densities where quark degrees of freedom become important. However, because of the large radii and the picture of overlapping bags it is not clear how far the equation of state can be extrapolated into the high-density regime.

V. SUMMARY

In this paper we study properties of nuclear matter based on an improved quark-meson coupling model. This model describes nucleons as nonoverlapping MIT bags interacting through scalar and vector mean fields. Of central importance is the bag constant which we assume to depend on the density of the nuclear environment. We study two types of models for the bag constant in which the density dependence is parametrized in terms of in-medium quantities. In one we assume that the bag constant depends on the scalar field only and in another the density dependence is related to the effective nucleon mass.

By performing a redefinition of the scalar field we demonstrate that the resulting energy functional corresponds to a QHD-type hadronic mean-field model with a general nonlinear scalar potential. In principle, this connection can be used to generate hadronic potentials from quark models. Moreover, a direct relation between nuclear phenomenology and the quark picture arises. We use this connection to motivate our models for the density dependence of the bag constant.

For the explicit calculations we employ a polynomial and a Padé form to model the medium dependence of the bag constant. The unknown parameters can then be fit to properties of nuclear matter near equilibrium that are known to be characteristic of the observed bulk and single-particle properties of nuclei.

Our basic goal is to study properties of nuclear matter. We investigate whether the models for the bag constant lead to results which are consistent with established hadronic

models. This is relevant in view of the hope to apply quark models to describe “new” physics which goes beyond the hadronic picture. Because of the relation between MQMC and QHD we compare our results with a QHD model calibrated to produce the same equilibrium properties. As the basic result we find an excellent agreement between our refined MQMC models and QHD at low and moderate densities. In particular it is possible to reproduce the desired saturation properties and large scalar and vector potentials as demanded by nuclear phenomenology. The central quantity here is the effective nucleon mass. Its accurate reproduction requires a rapidly decreasing bag constant below nuclear matter densities which leads to a satisfactory saturation mechanism. In contrast, the original QMC model predicts a very high effective nucleon mass and saturation is achieved on much smaller energy scales.

Moreover, the calibration procedure determines the value of the bag constant as a function of the effective nucleon mass in a model independent manner. However, this does not imply that the density dependence of B is well constrained. On the contrary, we find that models which reproduce identical properties of nuclear matter can be generated from different parametrizations and models for the bag constant. In particular there is no clear evidence for a specific scaling behavior of B .

We observe a similar model dependence in the predicted bag radius. As a consequence of the decreasing bag constant the size of the nucleon significantly increases in all our models. This effect is even more drastic than in earlier calculations [11,12]. We find that the nucleon bags start overlapping at densities slightly above the saturation point signaling the break down of the model. Moreover, different models and parametrizations which are equivalent near equilibrium produce different high-density equations of states. A similar picture arises in hadronic mean-field models [24]. Thus, even if quark degrees of freedom are incorporated it is unclear how far the nuclear equation of state can be extrapolated into the high-density regime.

In recent applications the original QMC model was used to estimate the density dependence of quark condensates and other hadron masses [8]. In view of our results we expect that the predictions for these quantities change drastically if the density dependence of the

bag constant is taken into account. It is also important to investigate the uncertainties in these quantities which arise from the model and parameter dependence of the bag constant.

In summary we conclude that an improved QMC model provides a very satisfactory description of nuclear matter near equilibrium. Phenomenological information in terms of equilibrium properties of nuclear matter can be used to constrain the unknown density dependence of the bag constant. On the hadronic level different models predict equivalent equation of states at low and moderate densities. This picture changes if microscopic quantities on the quark level are considered. Here a clear model and parameter dependence emerges. The difficulties rely on the detailed density dependence of the bag constant. Although these details have minor impact on the nuclear matter properties, microscopic quantities like the nucleon radius appear to be very sensitive. It is important to have these uncertainties under control before one can make reliable statements about the physics beyond the standard hadronic picture. To achieve this goal more detailed empirical information on effective hadron masses and also additional observables from finite nuclei might provide useful constraints to reduce model and parameter dependence in the future.

ACKNOWLEDGMENTS

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TABLES

TABLE I. Equilibrium Properties of Nuclear Matter

$(k_F)^0$	ρ_N^0	$M_{N,0}^*/M_N^*$	e_0	K_0
1.3 fm^{-1}	0.1484 fm^{-3}	0.60	-16.1 MeV	250 MeV

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FIGURE CAPTIONS

FIG. 1. Normalized effective coupling $C_q(\sigma)$ for the model MQMC_A, calculated with $\kappa = 4$ and for various values of g_σ^q . The corresponding quantity for QHD and for the original QMC model ($B = \text{const.}$) is also indicated.

FIG. 2. Effective nucleon mass as a function of the density for the model MQMC_A. Results for various values of κ and g_σ^q are compared with QHD and the original QMC model ($B = \text{const.}$).

FIG. 3. Effective nucleon mass as a function of the density for the model MQMC_B. Results for different parameter sets are compared with QHD.

FIG. 4. Predicted nonlinear scalar potential as a function of the transformed scalar field $g_0\phi = M_N - M_N^*$. The dotted and dashed curves correspond to MQMC_A with $g_\sigma^q = 0$ and $g_\sigma^q = 1$ respectively. The dotted-dashed curves indicate the result for MQMC_B. In addition the result for the original QMC model ($B = \text{const.}$) and the QHD potential is also indicated.

FIG. 5. Binding energy as a function of the density for MQMC_A. The dotted and dashed curves are calculated for $\kappa = 3, 4, 5$ from the bottom to the top. We also show the binding energy for the original QMC model ($B = \text{const.}$) and for QHD.

FIG. 6. Kinetic energy \mathcal{E}^0/ρ_N , vector potential U_v/ρ_N and scalar potential U_s/ρ_N as a function of the density. Results for MQMC_A, QHD and the original QMC model ($B = \text{const.}$) are shown. The parameters for MQMC_A are $\kappa = 3$ and $g_\sigma^q = 1$.

FIG. 7. Bag constant as a function of the effective nucleon mass calculated for $\kappa = 4$ and for different quark-meson couplings g_σ^q . In part (a) we show the results for MQMC_A. Here the curve $(M_N^*/M_N)^4$ corresponds to $g_\sigma^q = 0$. Part (B) indicates the results for MQMC_B.

FIG. 8. Bag constant at the saturation point for MQMC_A as a function of the quark-meson coupling g_σ^q . The individual curves correspond to different values of the nucleon mass at equilibrium. The bag constant at equilibrium does not depend on κ and the compressibility.

FIG. 9. Bag radius as a function of the density. At the critical radius $R_c = (3/4\pi\rho_N)^{1/3}$ the individual nucleon bags start overlapping.

FIGURES

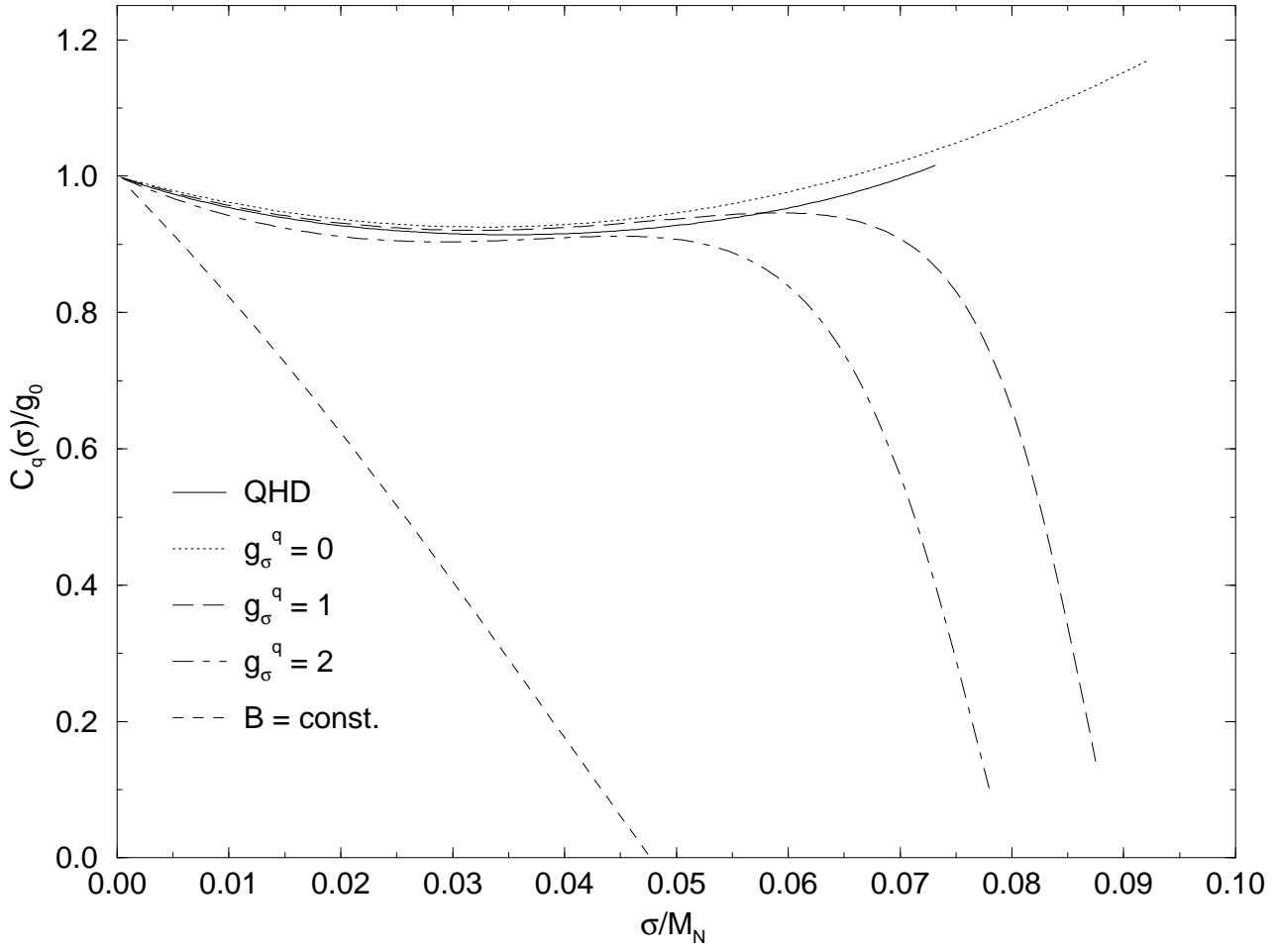


FIGURE 1

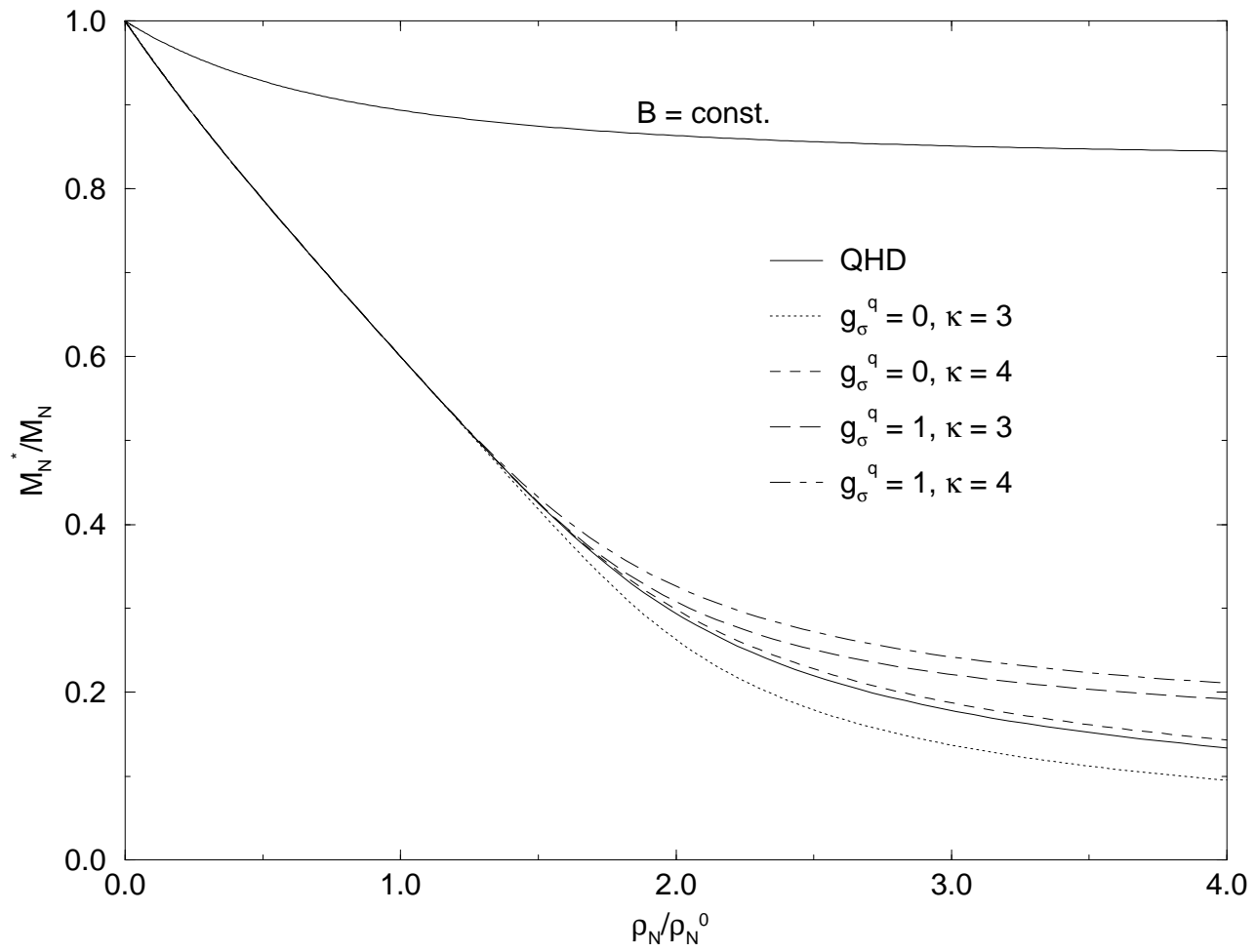


FIGURE 2

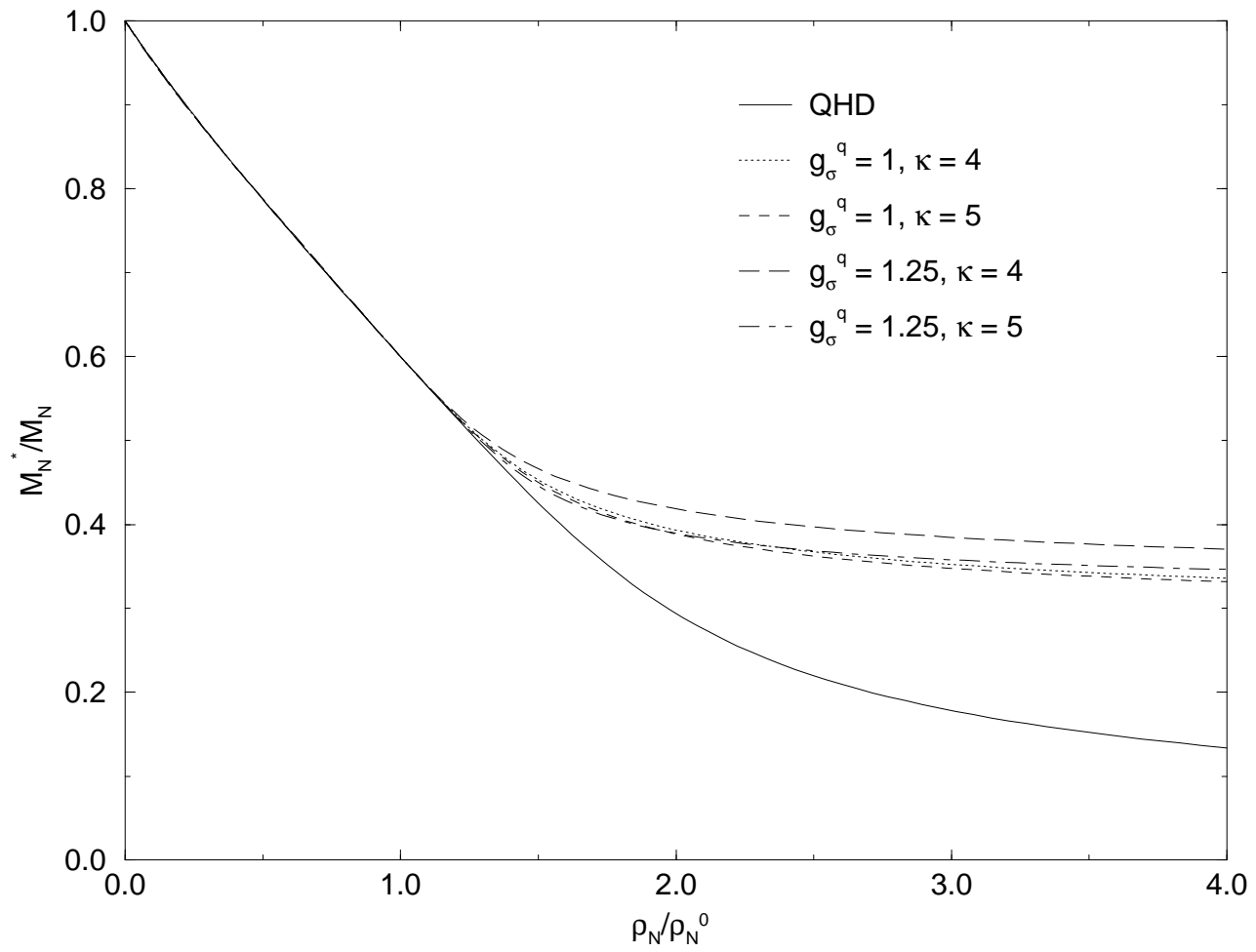


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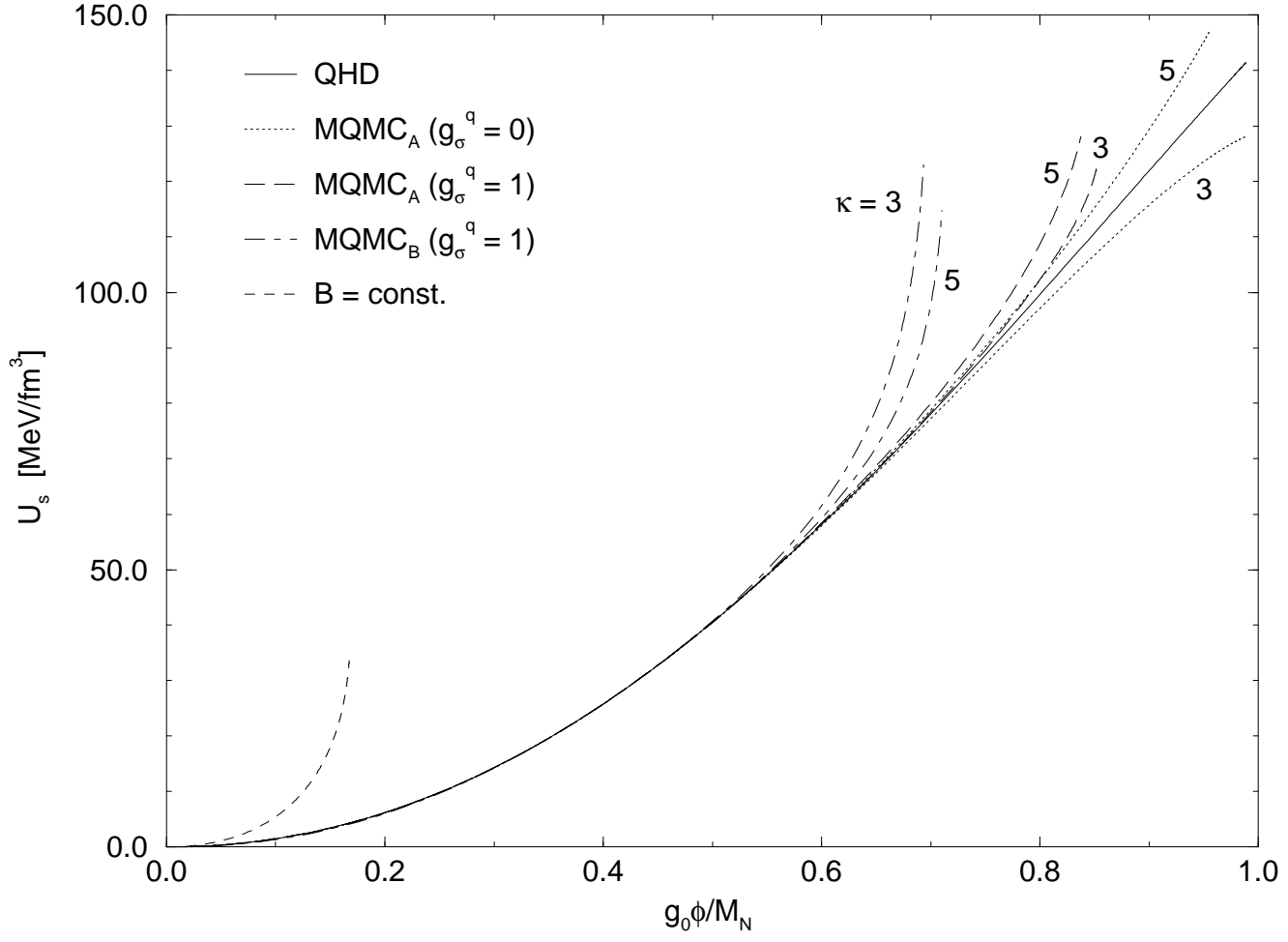


FIGURE 4

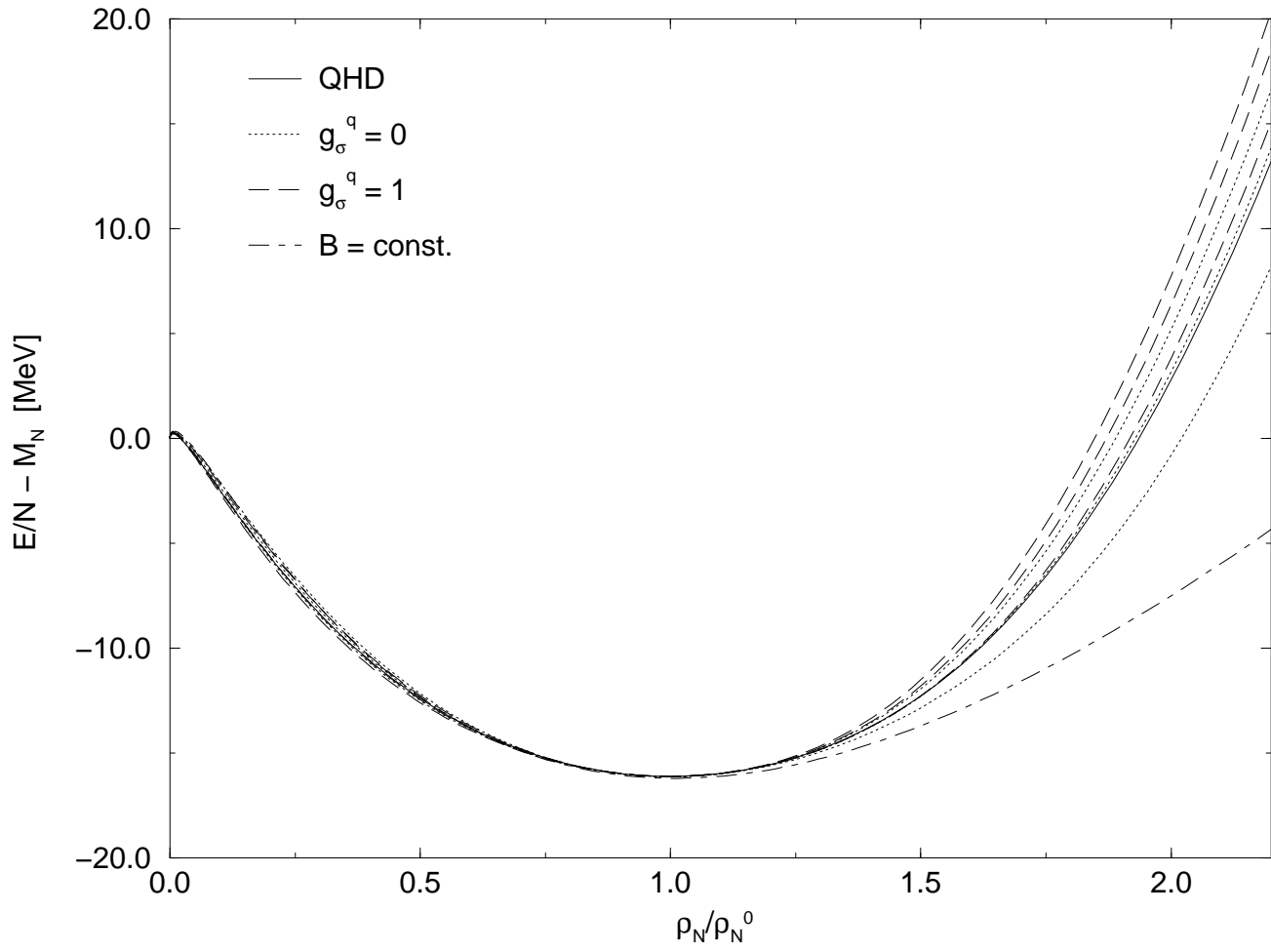


FIGURE 5

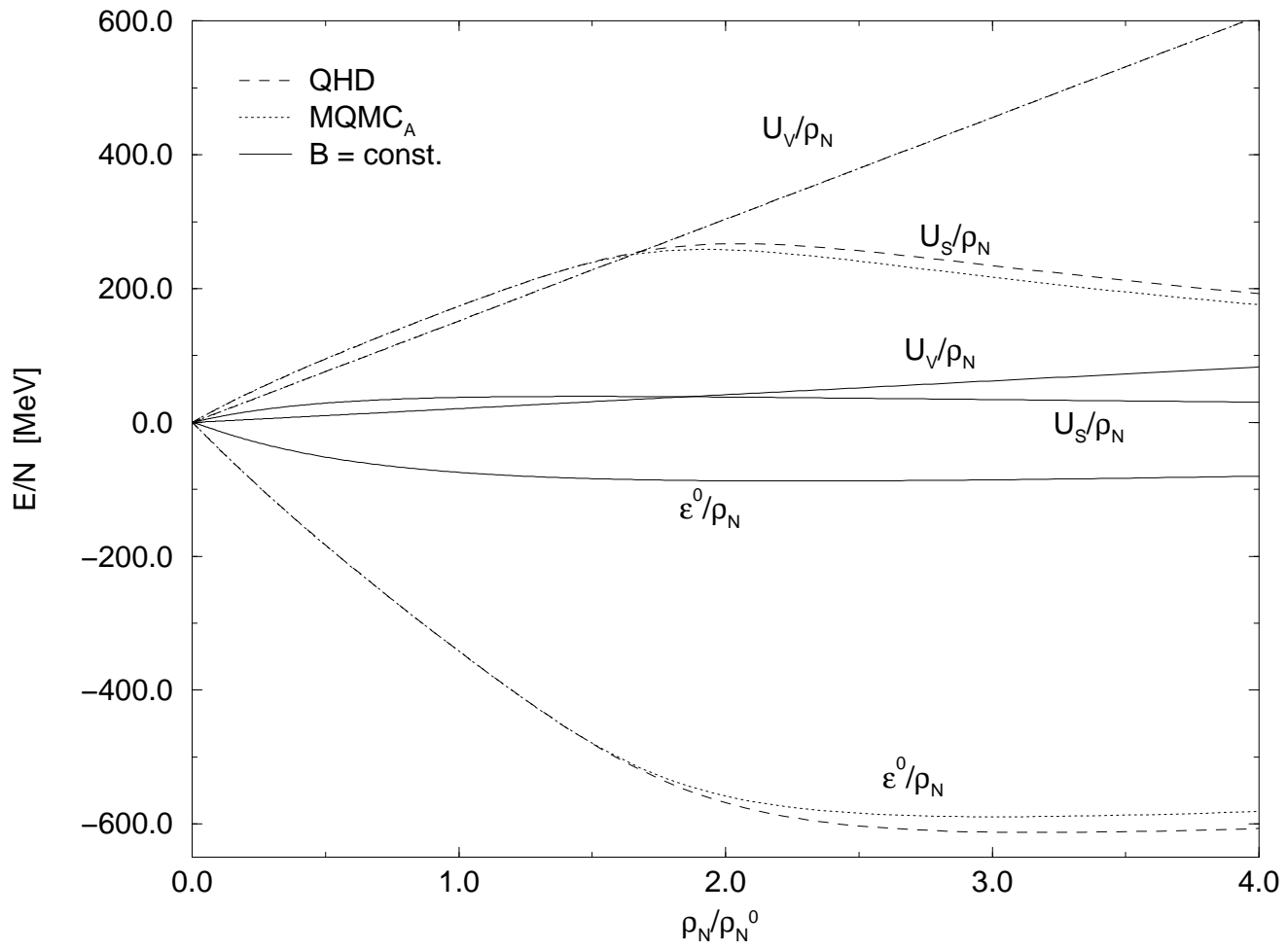


FIGURE 6

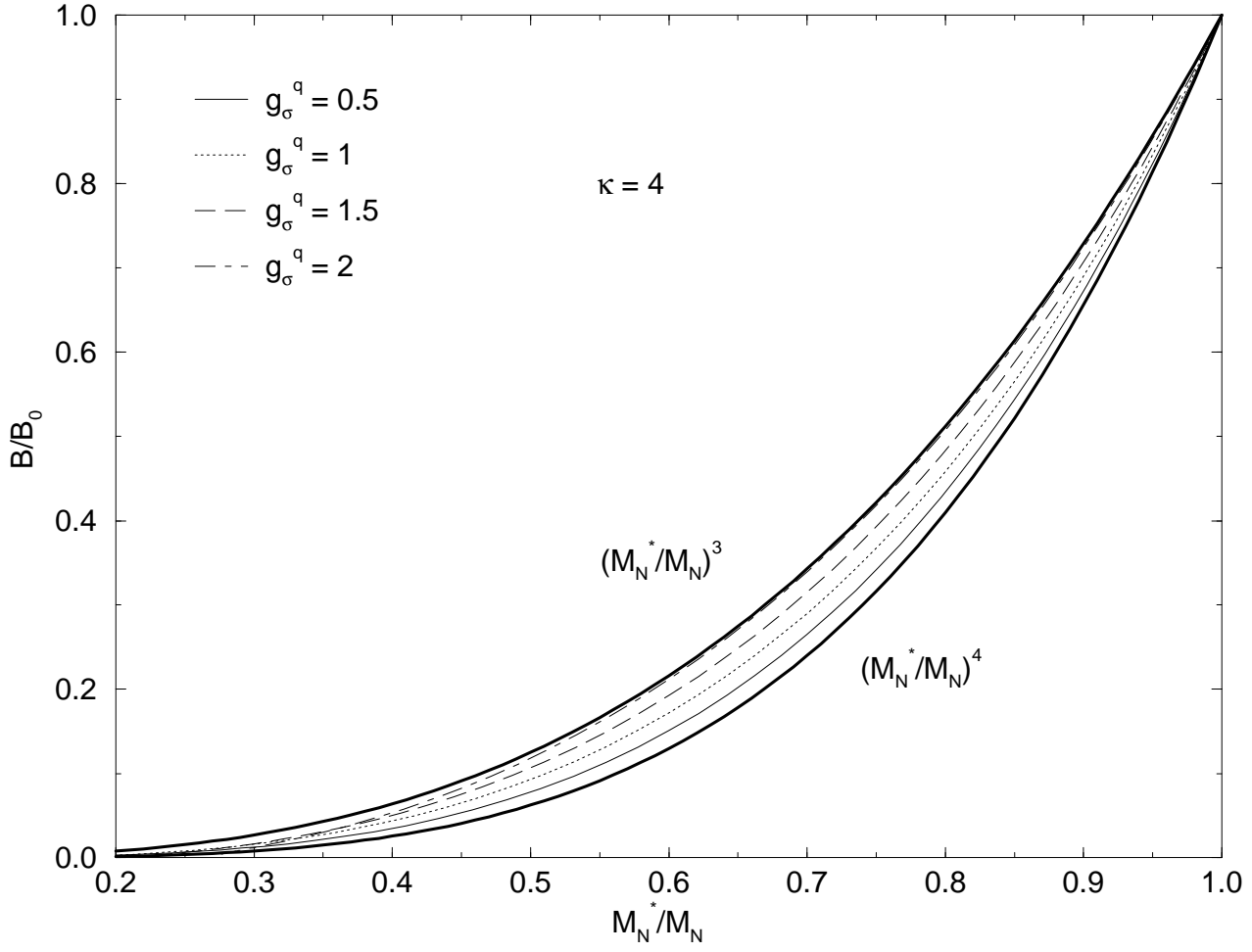


FIGURE 7a

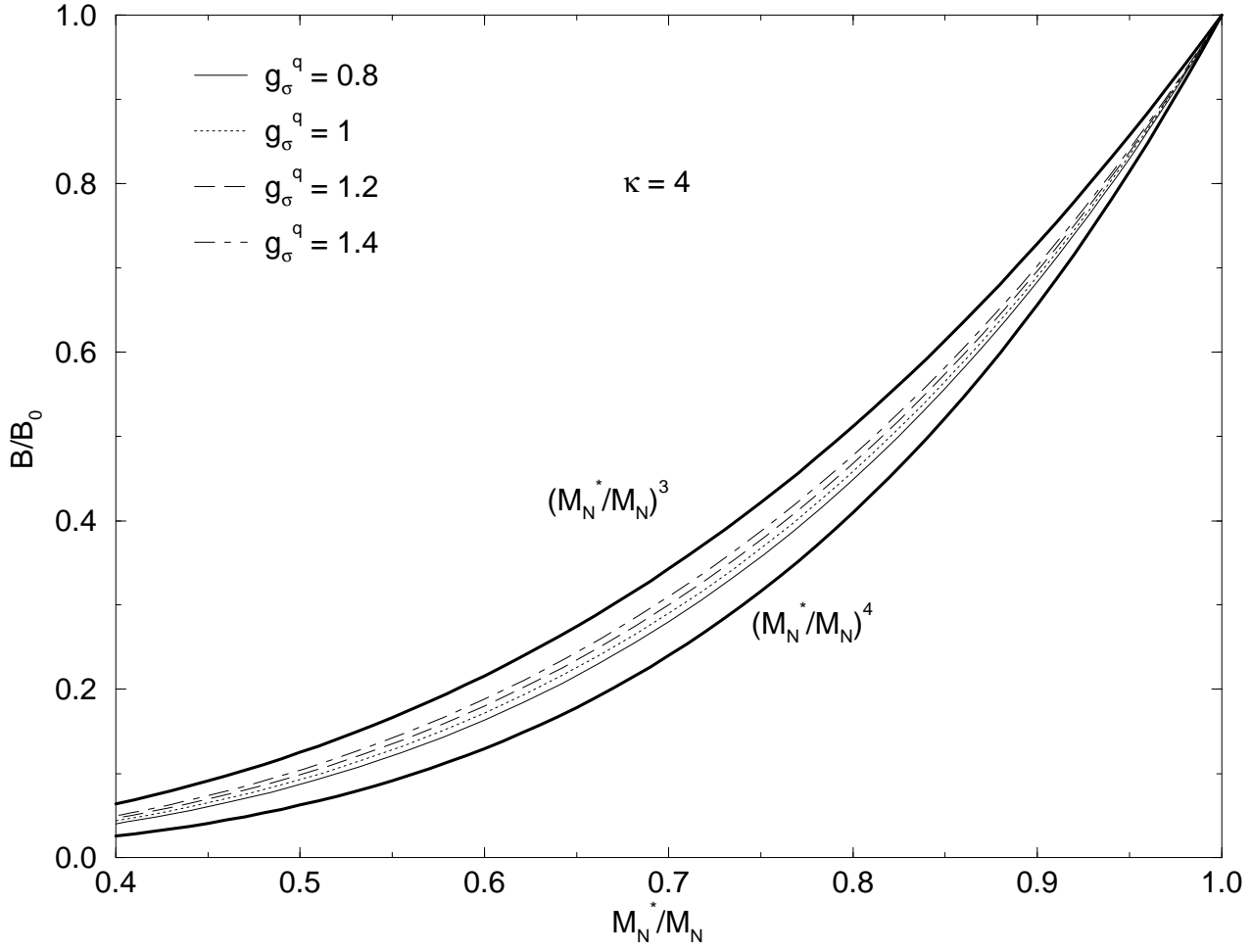


FIGURE 7b

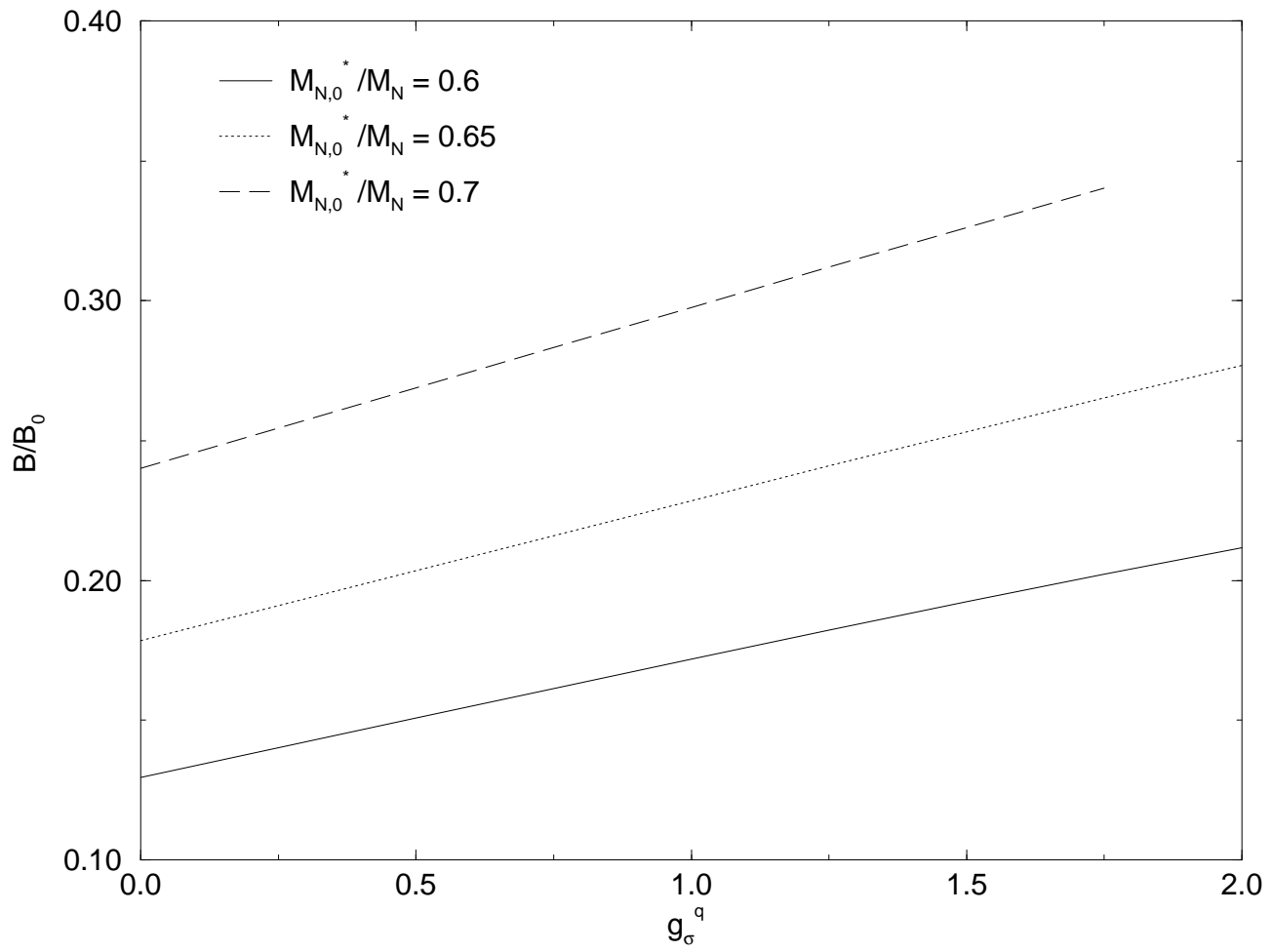


FIGURE 8

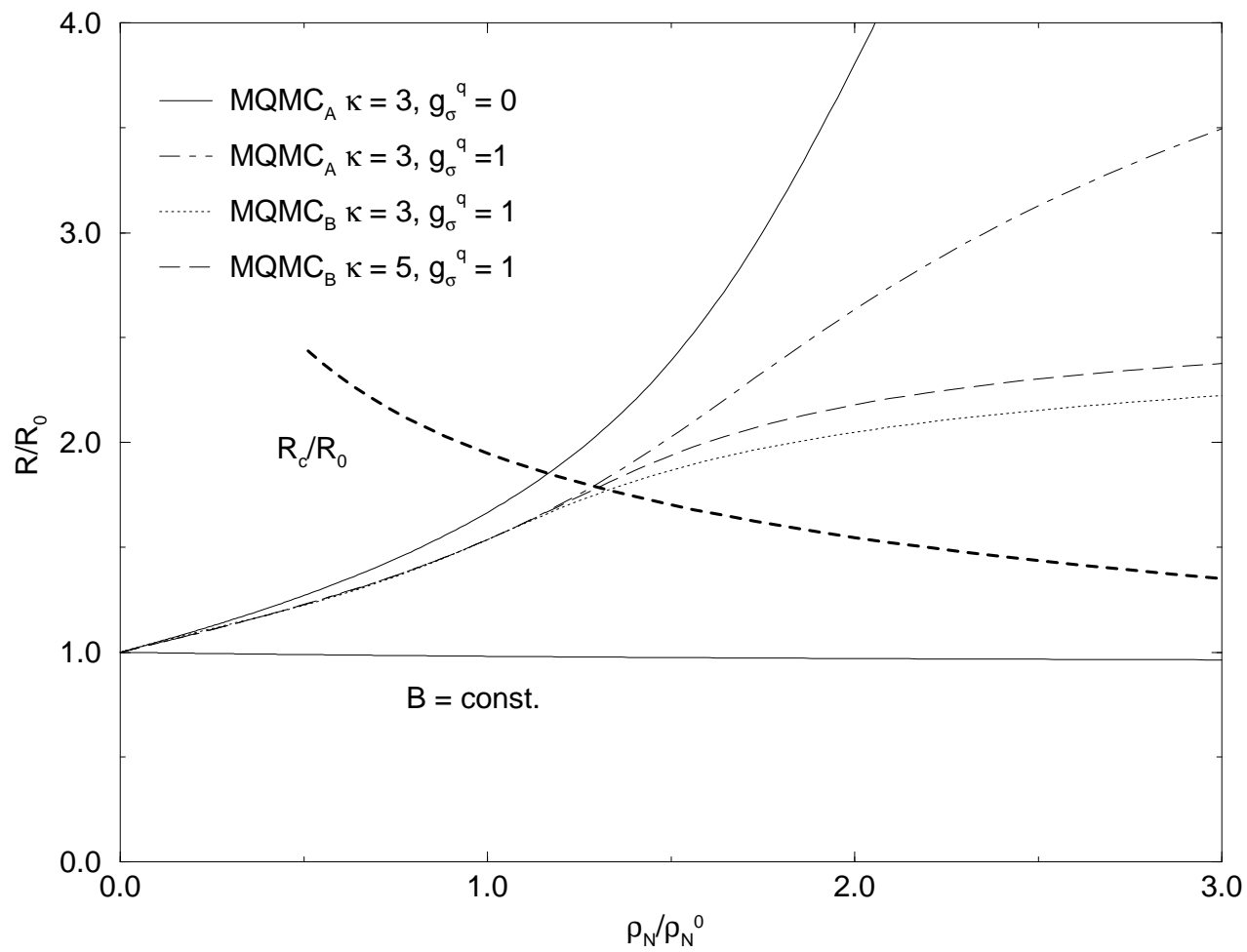


FIGURE 9